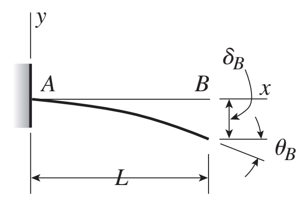
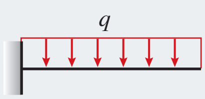
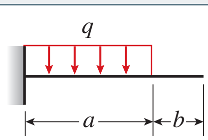
APPENDIX G

Deflections and Slopes of Beams

Table G-1

Deflections and Slopes of Cantilever Beams

		v = deflection in the y direction (positive upward) $v' = dv/dx$ = slope of the deflection curve $\delta_B = -v(L)$ = deflection at end B of the beam (positive downward) $\theta_B = -v'(L)$ = angle of rotation at end B of the beam (positive clockwise) EI = constant
1		$v = -\frac{qx^2}{24EI}(6L^2 - 4Lx + x^2)$ $\quad v' = -\frac{qx}{6EI}(3L^2 - 3Lx + x^2)$ $\delta_B = \frac{qL^4}{8EI}$ $\quad \theta_B = \frac{qL^3}{6EI}$
2		$v = -\frac{qx^2}{24EI}(6a^2 - 4ax + x^2)$ $\quad (0 \leq x \leq a)$ $v' = -\frac{qx}{6EI}(3a^2 - 3ax + x^2)$ $\quad (0 \leq x \leq a)$ $v = -\frac{qa^3}{24EI}(4x - a)$ $\quad v' = -\frac{qa^3}{6EI}$ $\quad (a \leq x \leq L)$ At $x = a$: $v = -\frac{qa^4}{8EI}$ $\quad v' = -\frac{qa^3}{6EI}$ $\delta_B = \frac{qa^3}{24EI}(4L - a)$ $\quad \theta_B = \frac{qa^3}{6EI}$

(*Continued*)

Table G-1 (Continued)

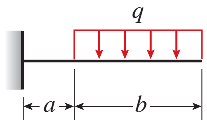
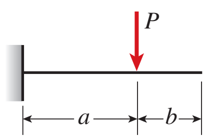
3		$v = -\frac{qbx^2}{12EI}(3L + 3a - 2x) \quad (0 \leq x \leq a)$ $v' = -\frac{qbx}{2EI}(L + a - x) \quad (0 \leq x \leq a)$ $v = -\frac{q}{24EI}(x^4 - 4Lx^3 + 6L^2x^2 - 4a^3x + a^4) \quad (a \leq x \leq L)$ $v' = -\frac{q}{6EI}(x^3 - 3Lx^2 + 3L^2x - a^3) \quad (a \leq x \leq L)$ At $x = a$: $v = -\frac{qa^2b}{12EI}(3L + a) \quad v' = -\frac{qabL}{2EI}$ $\delta_B = \frac{q}{24EI}(3L^4 - 4a^3L + a^4) \quad \theta_B = \frac{q}{6EI}(L^3 - a^3)$
4		$v = -\frac{Px^2}{6EI}(3L - x) \quad v' = -\frac{Px}{2EI}(2L - x)$ $\delta_B = \frac{PL^3}{3EI} \quad \theta_B = \frac{PL^2}{2EI}$
5		$v = -\frac{Px^2}{6EI}(3a - x) \quad v' = -\frac{Px}{2EI}(2a - x) \quad (0 \leq x \leq a)$ $v = -\frac{Pa^2}{6EI}(3x - a) \quad v' = -\frac{Pa^2}{2EI} \quad (a \leq x \leq L)$ At $x = a$: $v = -\frac{Pa^3}{3EI} \quad v' = -\frac{Pa^2}{2EI}$ $\delta_B = \frac{Pa^2}{6EI}(3L - a) \quad \theta_B = \frac{Pa^2}{2EI}$
6		$v = -\frac{M_0x^2}{2EI} \quad v' = -\frac{M_0x}{EI}$ $\delta_B = \frac{M_0L^2}{2EI} \quad \theta_B = \frac{M_0L}{EI}$

Table G-1 (Continued)

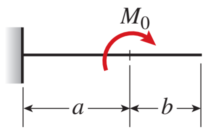
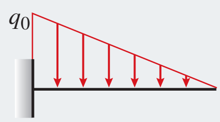
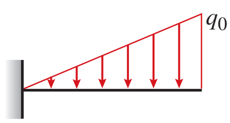
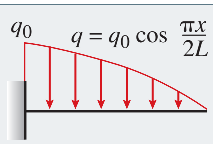
7		$v = -\frac{M_0x^2}{2EI} \quad v' = -\frac{M_0x}{EI} \quad (0 \leq x \leq a)$ $v = -\frac{M_0a}{2EI}(2x - a) \quad v' = -\frac{M_0a}{EI} \quad (a \leq x \leq L)$ At $x = a$: $\quad v = -\frac{M_0a^2}{2EI} \quad v' = -\frac{M_0a}{EI}$ $\delta_B = \frac{M_0a}{2EI}(2L - a) \quad \theta_B = \frac{M_0a}{EI}$
8		$v = -\frac{q_0x^2}{120LEI}(10L^3 - 10L^2x + 5Lx^2 - x^3)$ $v' = -\frac{q_0x}{24LEI}(4L^3 - 6L^2x + 4Lx^2 - x^3)$ $\delta_B = \frac{q_0L^4}{30EI} \quad \theta_B = \frac{q_0L^3}{24EI}$
9		$v = -\frac{q_0x^2}{120LEI}(20L^3 - 10L^2x + x^3)$ $v' = -\frac{q_0x}{24LEI}(8L^3 - 6L^2x + x^3)$ $\delta_B = \frac{11q_0L^4}{120EI} \quad \theta_B = \frac{q_0L^3}{8EI}$
10		$v = -\frac{q_0L}{3\pi^4EI}\left(48L^3 \cos\frac{\pi x}{2L} - 48L^3 + 3\pi^3Lx^2 - \pi^3x^3\right)$ $v' = -\frac{q_0L}{\pi^3EI}\left(2\pi^2Lx - \pi^2x^2 - 8L^2 \sin\frac{\pi x}{2L}\right)$ $\delta_B = \frac{2q_0L^4}{3\pi^4EI}(\pi^3 - 24) \quad \theta_B = \frac{q_0L^3}{\pi^3EI}(\pi^2 - 8)$

Table G-2

Deflections and Slopes of Simple Beams

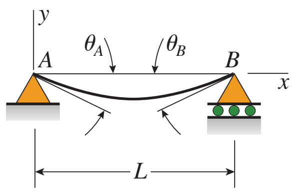
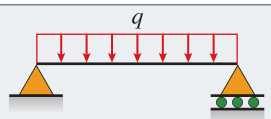
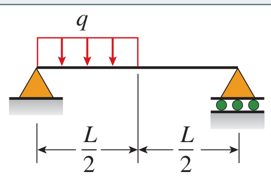
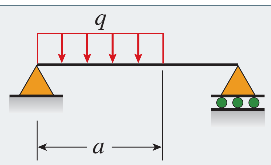
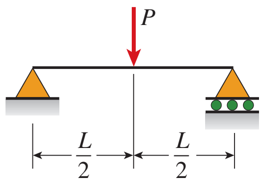
		v = deflection in the y direction (positive upward) $v' = dv/dx$ = slope of the deflection curve $\delta_C = -v(L/2)$ = deflection at midpoint C of the beam (positive downward) x_1 = distance from support A to point of maximum deflection $\delta_{\max} = -v_{\max}$ = maximum deflection (positive downward) $\theta_A = -v'(0)$ = angle of rotation at left-hand end of the beam (positive clockwise) $\theta_B = v'(L)$ = angle of rotation at right-hand end of the beam (positive counterclockwise) EI = constant
1		$v = -\dfrac{qx}{24EI}(L^3 - 2Lx^2 + x^3)$ $v' = -\dfrac{q}{24EI}(L^3 - 6Lx^2 + 4x^3)$ $\delta_C = \delta_{\max} = \dfrac{5qL^4}{384EI} \qquad \theta_A = \theta_B = \dfrac{qL^3}{24EI}$
2		$v = -\dfrac{qx}{384EI}(9L^3 - 24Lx^2 + 16x^3) \qquad \left(0 \le x \le \dfrac{L}{2}\right)$ $v' = -\dfrac{q}{384EI}(9L^3 - 72Lx^2 + 64x^3) \qquad \left(0 \le x \le \dfrac{L}{2}\right)$ $v = -\dfrac{qL}{384EI}(8x^3 - 24Lx^2 + 17L^2x - L^3) \qquad \left(\dfrac{L}{2} \le x \le L\right)$ $v' = -\dfrac{qL}{384EI}(24x^2 - 48Lx + 17L^2) \qquad \left(\dfrac{L}{2} \le x \le L\right)$ $\delta_C = \dfrac{5qL^4}{768EI} \qquad \theta_A = \dfrac{3qL^3}{128EI} \qquad \theta_B = \dfrac{7qL^3}{384EI}$
3		$v = -\dfrac{qx}{24LEI}(a^4 - 4a^3L + 4a^2L^2 + 2a^2x^2 - 4aLx^2 + Lx^3) \qquad (0 \le x \le a)$ $v' = -\dfrac{q}{24LEI}(a^4 - 4a^3L + 4a^2L^2 + 6a^2x^2 - 12aLx^2 + 4Lx^3) \qquad (0 \le x \le a)$ $v = -\dfrac{qa^2}{24LEI}(-a^2L + 4L^2x + a^2x - 6Lx^2 + 2x^3) \qquad (a \le x \le L)$ $v' = -\dfrac{qa^2}{24LEI}(4L^2 + a^2 - 12Lx + 6x^2) \qquad (a \le x \le L)$ $\theta_A = \dfrac{qa^2}{24LEI}(2L - a)^2 \qquad \theta_B = \dfrac{qa^2}{24LEI}(2L^2 - a^2)$

Table G-2 (Continued)

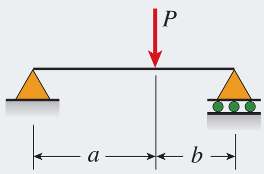
4



$$v = -\frac{Px}{48EI}(3L^2 - 4x^2) \qquad v' = -\frac{P}{16EI}(L^2 - 4x^2) \qquad \left(0 \le x \le \frac{L}{2}\right)$$

$$\delta_C = \delta_{max} = \frac{PL^3}{48EI} \qquad \theta_A = \theta_B = \frac{PL^2}{16EI}$$

5



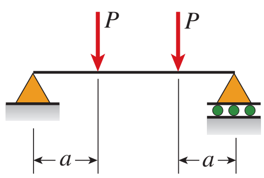
$$v = -\frac{Pbx}{6LEI}(L^2 - b^2 - x^2) \qquad v' = -\frac{Pb}{6LEI}(L^2 - b^2 - 3x^2) \qquad (0 \le x \le a)$$

$$\theta_A = \frac{Pab(L + b)}{6LEI} \qquad \theta_B = \frac{Pab(L + a)}{6LEI}$$

$$\text{If } a \ge b, \quad \delta_C = \frac{Pb(3L^2 - 4b^2)}{48EI} \qquad \text{If } a \le b, \quad \delta_C = \frac{Pa(3L^2 - 4a^2)}{48EI}$$

$$\text{If } a \ge b, \quad x_1 = \sqrt{\frac{L^2 - b^2}{3}} \quad \text{and} \quad \delta_{max} = \frac{Pb(L^2 - b^2)^{3/2}}{9\sqrt{3}LEI}$$

6



$$v = -\frac{Px}{6EI}(3aL - 3a^2 - x^2) \qquad v' = -\frac{P}{2EI}(aL - a^2 - x^2) \qquad (0 \le x \le a)$$

$$v = -\frac{Pa}{6EI}(3Lx - 3x^2 - a^2) \qquad v' = -\frac{Pa}{2EI}(L - 2x) \qquad (a \le x \le L - a)$$

$$\delta_C = \delta_{max} = \frac{Pa}{24EI}(3L^2 - 4a^2) \qquad \theta_A = \theta_B = \frac{Pa(L - a)}{2EI}$$

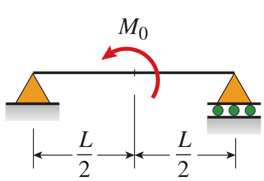
7

$$v = -\frac{M_0 x}{6LEI}(2L^2 - 3Lx + x^2) \qquad v' = -\frac{M_0}{6LEI}(2L^2 - 6Lx + 3x^2)$$

$$\delta_C = \frac{M_0 L^2}{16EI} \qquad \theta_A = \frac{M_0 L}{3EI} \qquad \theta_B = \frac{M_0 L}{6EI}$$

$$x_1 = L\left(1 - \frac{\sqrt{3}}{3}\right) \quad \text{and} \quad \delta_{max} = \frac{M_0 L^2}{9\sqrt{3}EI}$$

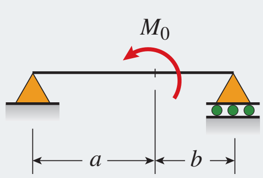
8



$$v = -\frac{M_0 x}{24LEI}(L^2 - 4x^2) \qquad v' = -\frac{M_0}{24LEI}(L^2 - 12x^2) \qquad \left(0 \le x \le \frac{L}{2}\right)$$

$$\delta_C = 0 \qquad \theta_A = \frac{M_0 L}{24EI} \qquad \theta_B = -\frac{M_0 L}{24EI}$$

9



$$v = -\frac{M_0 x}{6LEI}(6aL - 3a^2 - 2L^2 - x^2) \qquad (0 \le x \le a)$$

$$v' = -\frac{M_0}{6LEI}(6aL - 3a^2 - 2L^2 - 3x^2) \qquad (0 \le x \le a)$$

$$\text{At } x = a\text{:} \quad v = \frac{M_0 ab}{3LEI}(2a - L) \qquad v' = -\frac{M_0}{3LEI}(3aL - 3a^2 - L^2)$$

$$\theta_A = \frac{M_0}{6LEI}(6aL - 3a^2 - 2L^2) \qquad \theta_B = \frac{M_0}{6LEI}(3a^2 - L^2)$$

(*Continued*)

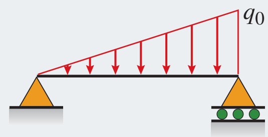
Table G-2 (Continued)

10

$$v = -\frac{M_0 x}{2EI}(L - x) \qquad v' = -\frac{M_0}{2EI}(L - 2x)$$

$$\delta_C = \delta_{max} = \frac{M_0 L^2}{8EI} \qquad \theta_A = \theta_B = \frac{M_0 L}{2EI}$$

11



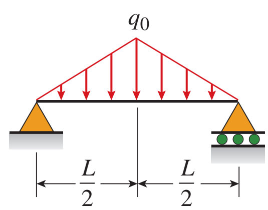
$$v = -\frac{q_0 x}{360LEI}(7L^4 - 10L^2x^2 + 3x^4)$$

$$v' = -\frac{q_0}{360LEI}(7L^4 - 30L^2x^2 + 15x^4)$$

$$\delta_C = \frac{5q_0L^4}{768EI} \qquad \theta_A = \frac{7q_0L^3}{360EI} \qquad \theta_B = \frac{q_0L^3}{45EI}$$

$$x_1 = 0.5193L \qquad \delta_{max} = 0.00652\frac{q_0L^4}{EI}$$

12

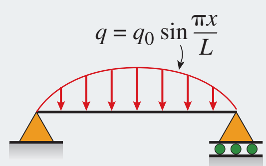


$$v = -\frac{q_0 x}{960LEI}(5L^2 - 4x^2)^2 \qquad \left(0 \le x \le \frac{L}{2}\right)$$

$$v' = -\frac{q_0}{192LEI}(5L^2 - 4x^2)(L^2 - 4x^2) \qquad \left(0 \le x \le \frac{L}{2}\right)$$

$$\delta_C = \delta_{max} = \frac{q_0L^4}{120EI} \qquad \theta_A = \theta_B = \frac{5q_0L^3}{192EI}$$

13



$$v = -\frac{q_0L^4}{\pi^4EI}\sin\frac{\pi x}{L} \qquad v' = -\frac{q_0L^3}{\pi^3EI}\cos\frac{\pi x}{L}$$

$$\delta_C = \delta_{max} = \frac{q_0L^4}{\pi^4EI} \qquad \theta_A = \theta_B = \frac{q_0L^3}{\pi^3EI}$$